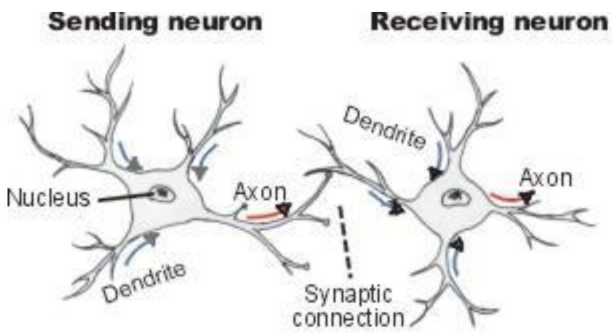
Artificial Neural Networks

Outline

- Brains
- Neural networks
- Perceptrons
- Multilayer perceptrons
- Applications of neural networks

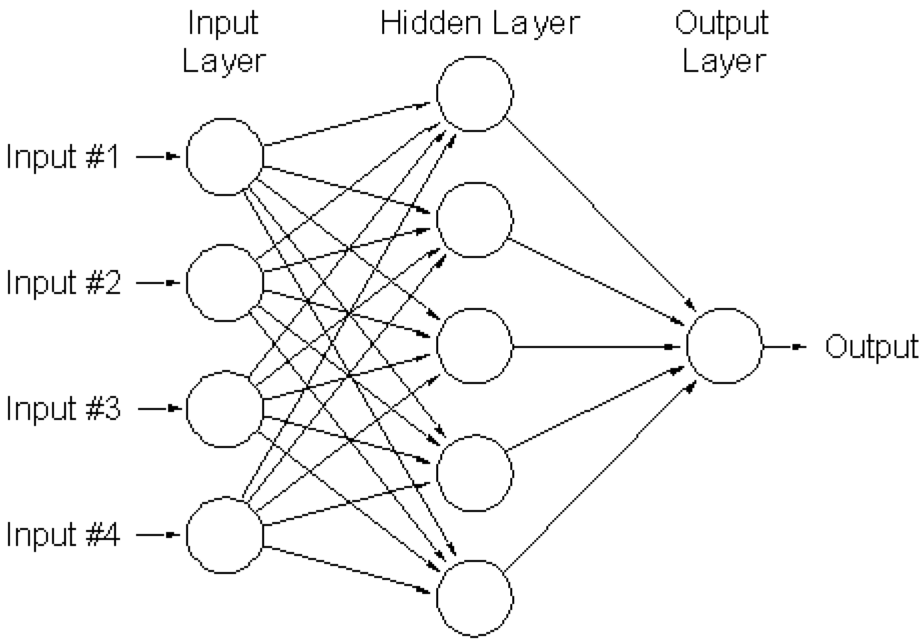
Introduction

- Artificial neural networks (ANNs) are patterned after the structure and function of the brain
- When a neuron fires, it sends an electro-chemical signal along its axon to the synapses which connect it to other neurons
  - If this signal is strong enough, the next neuron may also fire, resulting in a spreading activation pattern
  - The strength of the connections between neurons can change over time, and this is the basis for learning
    - Connections leading to a “good answer” are strengthened while those leading to a “bad answer” are weakened
  - Humans have about 10 billion neurons and 60 trillion synapses



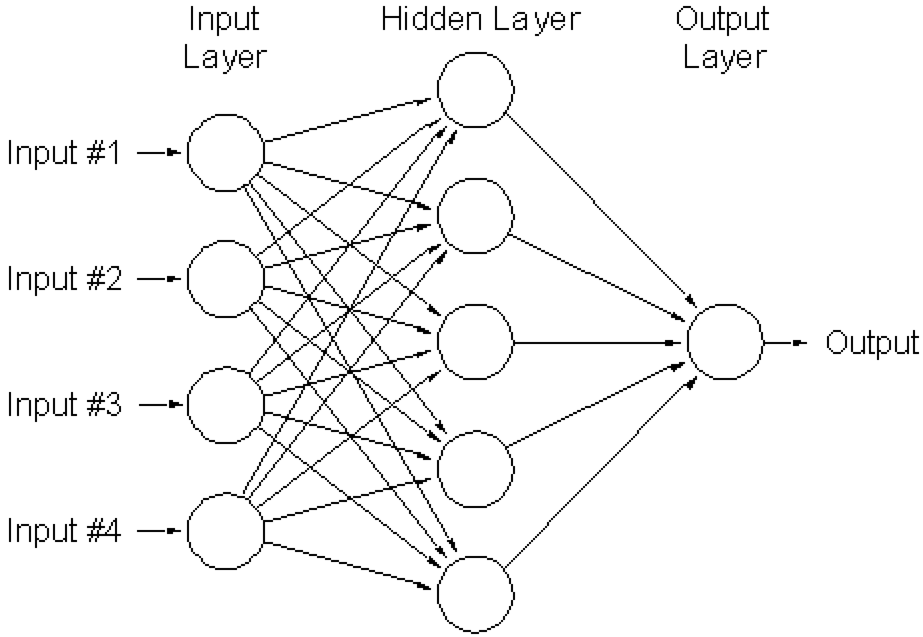
Introduction

- Artificial neural networks are patterned after the brain
  - Neurodes (or just nodes) represent neurons
  - Connections represent synapses
  - Weights on the connections change in order to produce learning



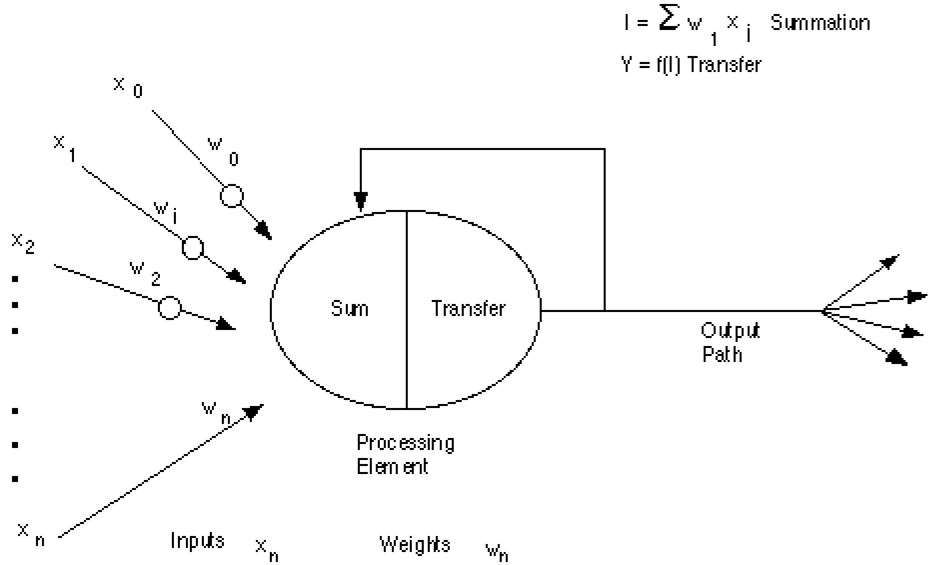
Introduction

- Architecture:
- In most cases, we use a fully connected model
  - All neurodes at one layer are connected to each of the neurodes at the next layer
  - This picture shows a fully connected model



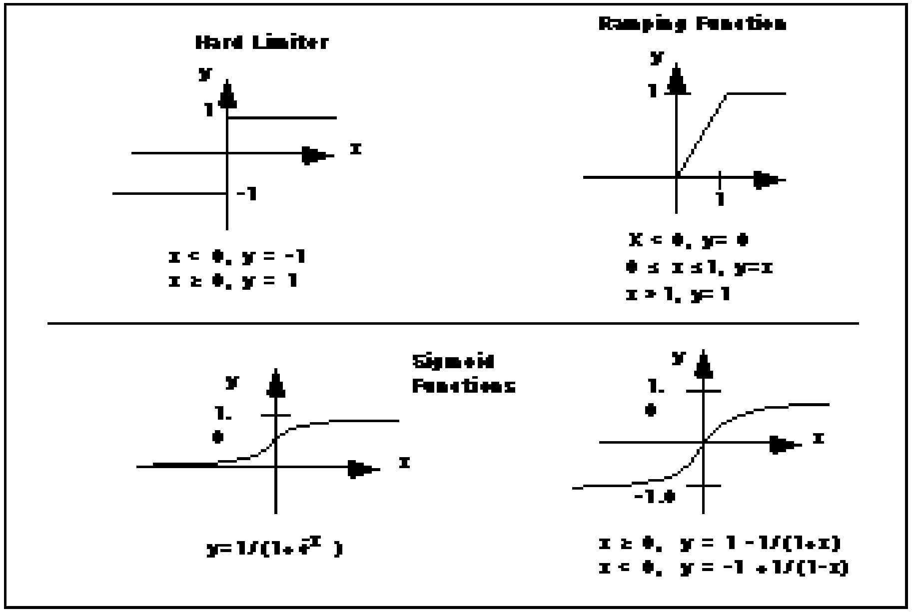
Introduction

- Each neurode sums the input signals coming into it
  - Actually, multiply the connection weight and the incoming signal, and sum each of these
- Output or "transfer" function could be:
  - Step function
  - Sign function
  - Sigmoid function
  - Linear function



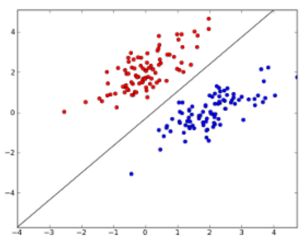
Introduction

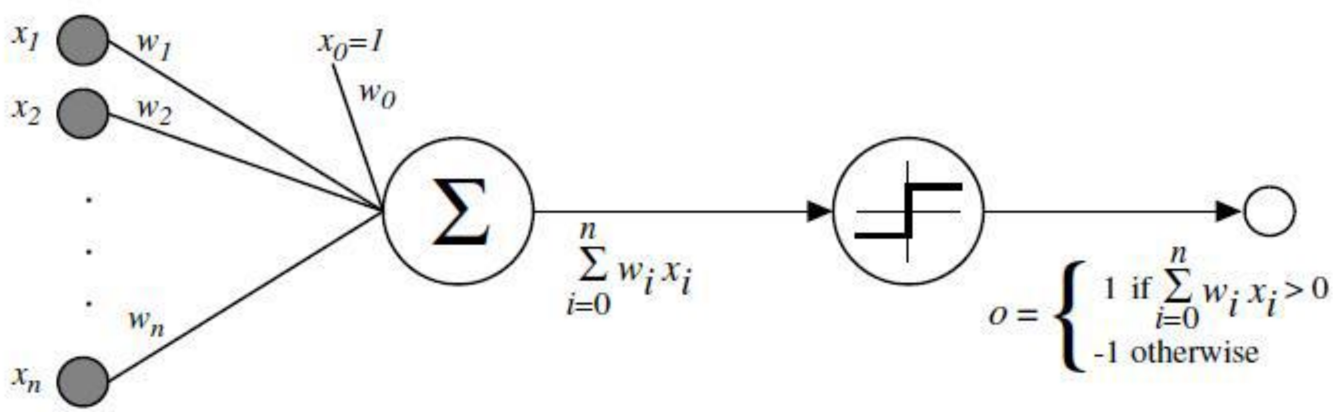
- Transfer functions
  - Step (or sign) function
    - “Hard Limiter”
  - Linear (ramping) function
  - Sigmoid function
    - Most common because it’s continuous
    - Usually used in backpropagation networks



- With two inputs, the decision boundary takes on the form of a straight line
  - So if you had a problem like this one, the perceptron could learn to solve it
    - “Linearly separable” (which extends beyond two dimensions)

Perceptron





Perceptron

- However, even very simple problems that are not linearly separable cannot be solved by a perceptron
  - e.g. Exclusive Or (XOR)

x_1 \ x_2	0	1
0	1	0
1	0	1

Multilayer Networks

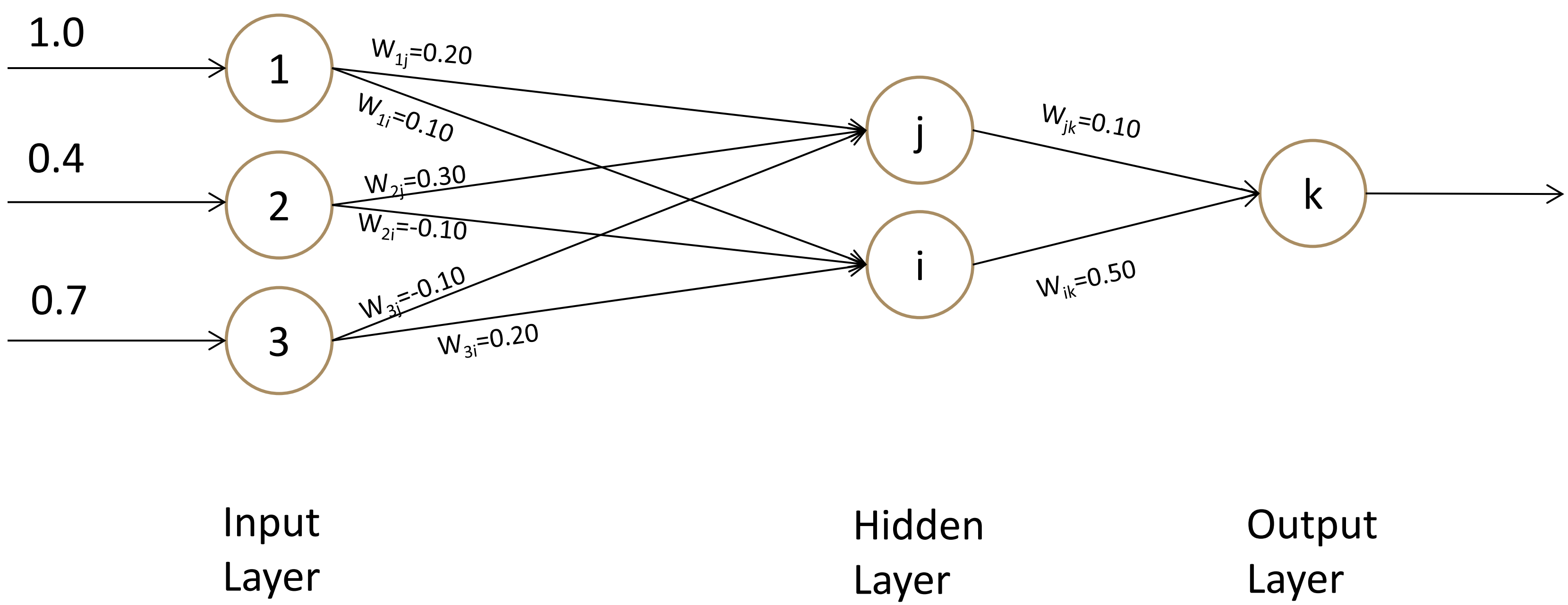
- Perceptron can’t solve problems that are not linearly separable, but a multilayer network can
- A multilayer network has one or more hidden layers between the input and output layers
- Usually a feed-forward, backpropagation architecture

Multilayer Networks

- Feed Forward:
  - Input to neuron is still $x_j = \sum_{i=1}^{n} x_i w_{i,j}$
    - n= number of connected inputs
    - x_i = the input on connection i
    - $w_{i,j}$ = the weight on the connection between neurode i and neurode j
  - Transfer function is sigmoid
    $$y_j = \frac{1}{1+e^{-x_j}}$$
    - This bounds the output between 0 and 1 and is continuously differentiable

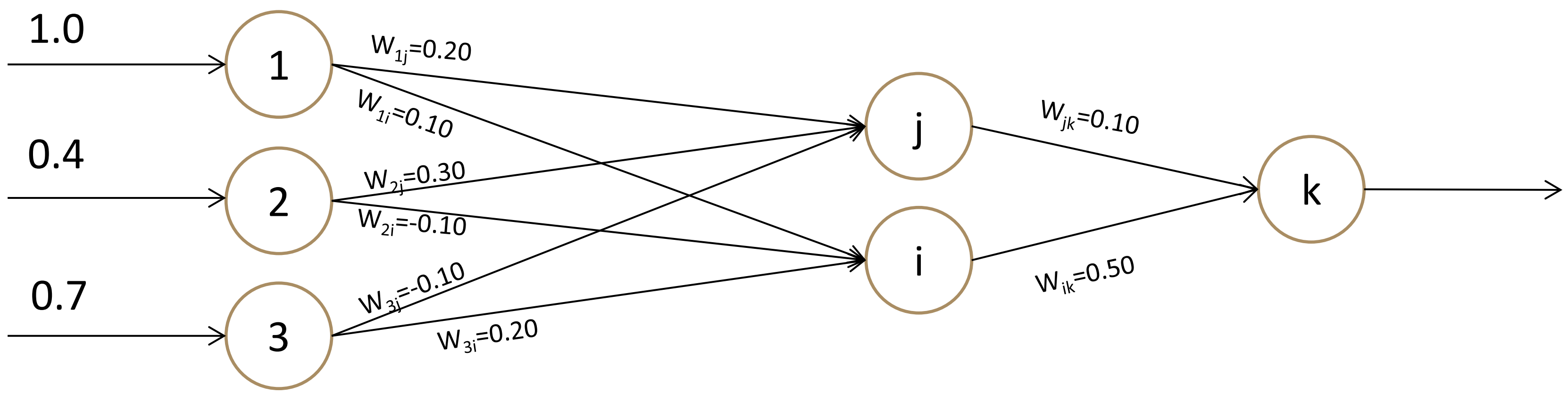
Multilayer Networks

- Feed forward example



- Feed forward example

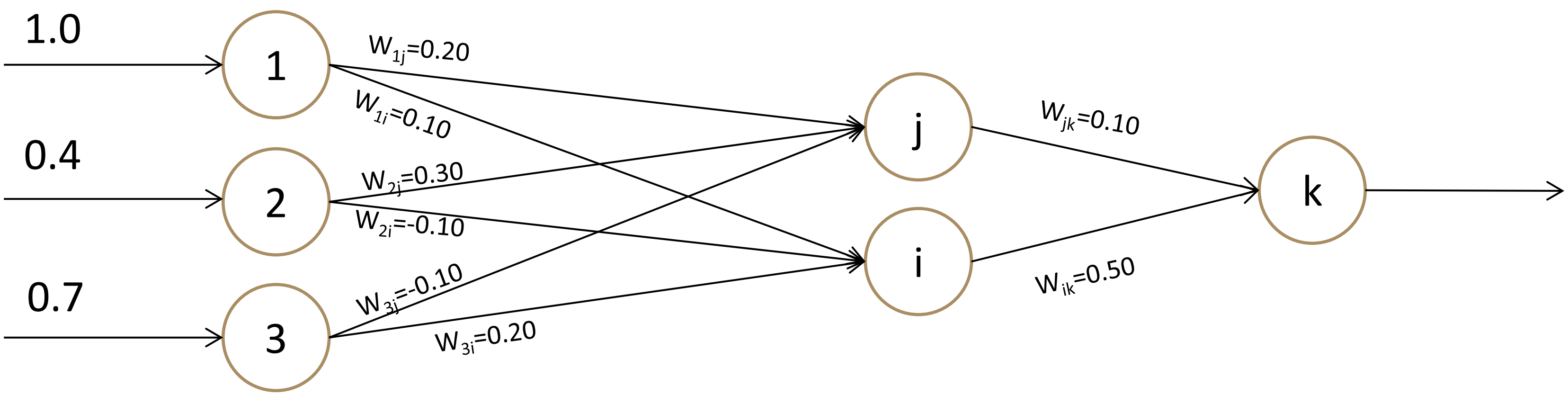
Multilayer Networks



- Input to node j = $\sum_{n=1}^{3} w_{n,j}\, o_n$ where o_n = output of node n
- Input to node j = w1j*1.0 + w2j*0.4 + w3j*0.7
  - = 0.2*1.0 + 0.3*0.4 + -0.1*0.7
  - =0.2 + 0.12 + -0.07 = 0.25

Multilayer Networks

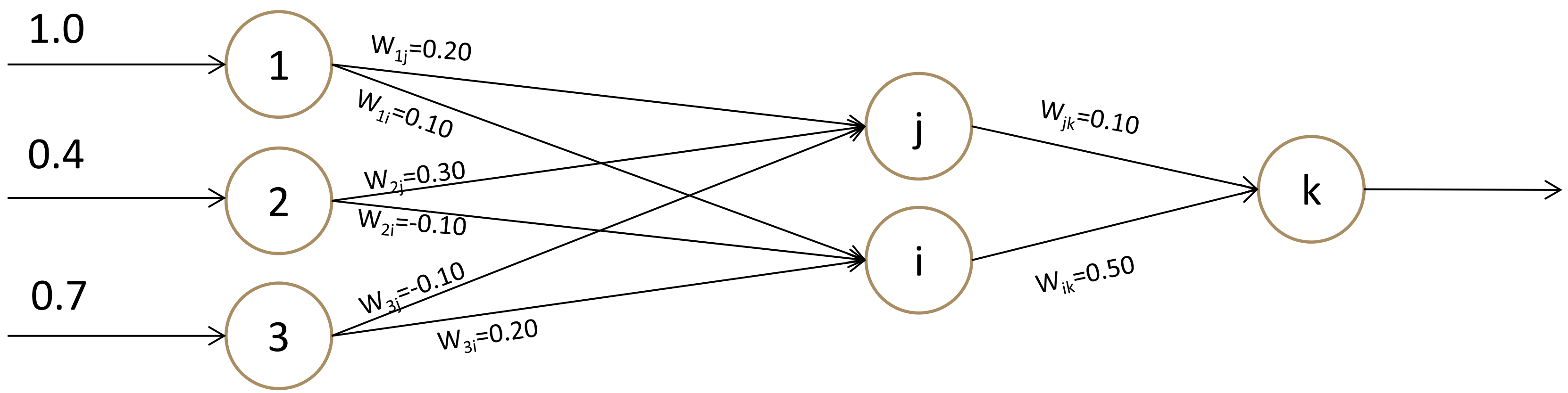
- Feed forward example



- Output from node j = $\frac{1}{1+e^{-input}}$ = 0.562177

- Feed forward example

Multilayer Networks



- Input to node j = 0.25, Output from node j = 0.562
- Input to node i = 0.20, Output from node i = 0.550
- Input to node k = 0.331, Output from node k = 0.582

Multilayer Networks

- Backpropagation:
  - Error at node j:
    - $Error(j) = (\sum_k Error(k) * w_{j,k}) * f'(x_j)$
    - Error(k) = output error at node k
    - w_{jk} = weight of connection between nodes j and k
    - $f'(x) = O_j (1-O_j)$
    - O_j = output at node j

Multilayer Networks

- Backpropagation:
  - The Delta Rule:
    - $w_{jk}(new) = w_{jk}(current) + \Delta w_{jk}$
    - $\Delta w_{jk} = r * Error(k) * O_j$
    - r = learning rate, $0 < r < 1$
    - Error(k) = error at node k
    - O_j = output of node j

Multilayer Networks

- Backpropagation Example:
  - $Error(j) = (\sum_k Error(k) * w_{j,k}) * f'(x_j)$
  - Let's say we want 0.599 as our output, so Error(k) is 0.017
  - Error(j) = 0.017 * 0.10 * 0.25 = 0.00042
    - w_{jk}(new) = w_{jk}(current) + Δw_{jk}
    - Δw_{jk} = r * Error(k) * O_j
  - Let's say our learning rate, r = 0.5
  - Δw_{jk} = 0.5 * 0.017 * 0.562 = 0.0048
  - w_{jk}(new) = 0.10 + 0.0048 = 0.1048

Steps in Training a Network:

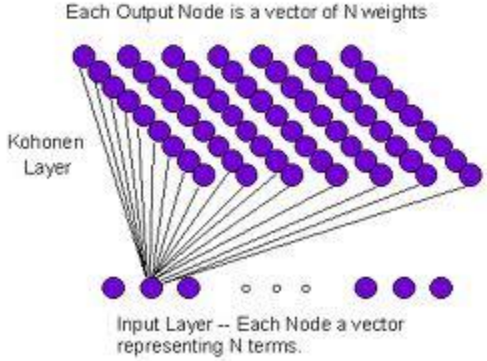
- Initialization
  - Randomly initialize weights between [-0.5, 0.5]
- Activation
  - Apply inputs $x_1 \ldots x_n$ and calculate the output
    - First, summation function
    - Then, transfer function – step function or sign function for perceptron, sigmoid most likely for multilayer
- Weight Adjustment
  - If the output is not what was desired, go back and adjust each weight
    - First, error function
    - Then, Delta rule
- Iterate until the error rate is acceptable (or we reach some other stopping condition)

Kohonen Self-Organizing Maps

- Unsupervised (!) neural network
- Competitive learning
  - Only a single output node is active for a given input
    - Winner takes all
- Kohonen's "principle of topographic map formation"
  - The spatial location of an active output neurode in the topographic map corresponds to a specific feature of the input pattern

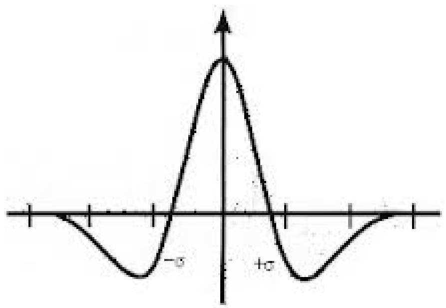
Kohonen Self-Organizing Maps

- Architecture / Behavior
  - Two layers – input and output (Kohonen layer)
  - Many more nodes in output layer than in input
  - Input layer is fully connected to the output layer
  - One input node for each input feature (attribute)



Kohonen Self-Organizing Maps

- Training / Learning
  - Input instances are presented to the input layer and fed through to the output layer
  - The single output node whose weights most closely match those of the input is the one that “wins”
  - The winner is rewarded by having its weights changed to match the input even more closely
  - Initially, those output neurodes near the winner are also rewarded
    - Size of “neighborhood” decreased as number of iterations increase
    - Mexican hat function
    - Neighborhood defined by city block or Euclidean distance
  - Output nodes winning the most instances during the last pass of the data through the network are saved
    - The number of output nodes eventually saved corresponds to the number of “classes” found by the network



General Considerations (for all ANNs)

- Training and Testing
  - "Epoch" is one pass of all of the training instances through the neural network
  - Rule of thumb in supervised learning is to use 80% of the data for training and 20% for testing
    - Can apply similar rule to Kohonen maps
      - Build clustering / classification network with 80% of cases and then see how remaining 20% are classified
  - Usually use root mean squared (rms) error but could also use:
    - Absolute error
    - Mean squared error

General Considerations (for all ANNs)

- Conditioning the Input
  - Input must be numeric
  - Works best if in the range of [0, 1]

General Considerations (for all ANNs)

- Categorical Input Data:
  - Divide interval range into equal sized units
    - red -> 0.00
    - green -> 0.33
    - blue -> 0.67
    - yellow -> 1.00
    - Pitfall here is it implies some sort of ordering on the data that is just not true (red < green?)
  - Use additional input nodes
    - red -> 0, 0
    - green -> 0, 1
    - blue -> 1, 0
    - yellow -> 1, 1

General Considerations (for all ANNs)

- Numeric Input Data:
  - Normalize into [0, 1] range
  - new_value = (original_value – min)/(max – min)
- Output Strategies
  - Reverse numeric range to scale output to original (non-normalized) input

General Considerations (for all ANNs)

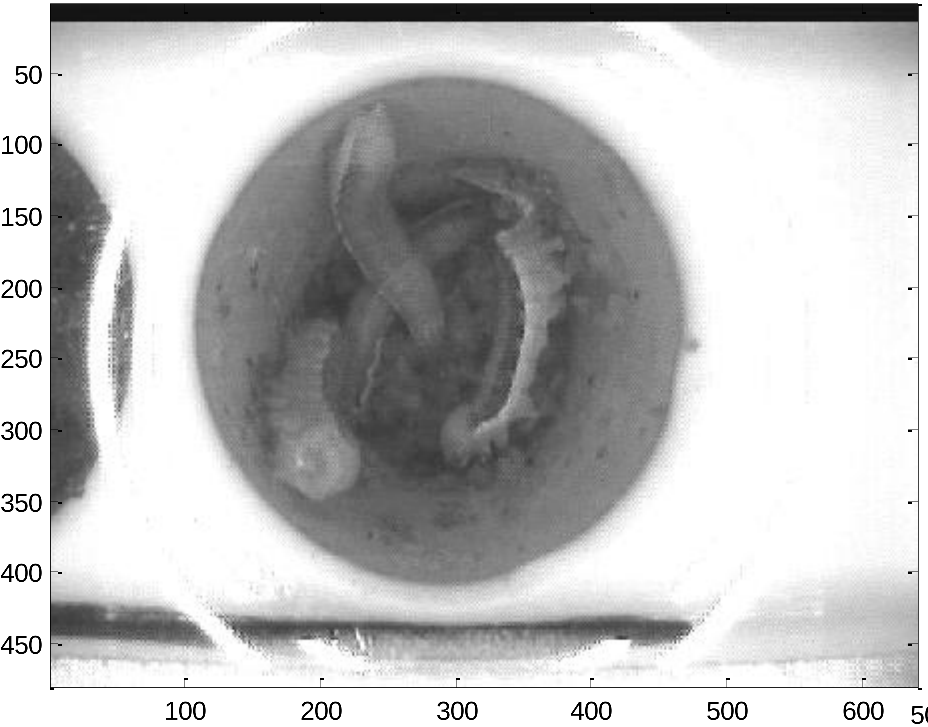
- Architecture
  - Input Layer
    - Number of nodes is equal to number of inputs
      - But, may vary these to get at your data better, particularly categorical data

General Considerations (for all ANNs)

- Architecture
  - Hidden Layers
    - Need to experiment with number of layers and number of nodes in each layer
    - Best is to use the least of each and still get convergence, but you need to figure out what “least” is
    - Too many nodes/layers, network will learn training data perfectly
      - Memorizes the training examples and doesn’t generalize
      - Overtraining
      - Does poorly on test data
    - Too few, won’t reach convergence
      - Can get oscillatory behavior on weight adjustments

General Considerations (for all ANNs)

- Architecture
  - Output Layer
    - Depends on what you want from the output
      - May choose to add more nodes for categorical output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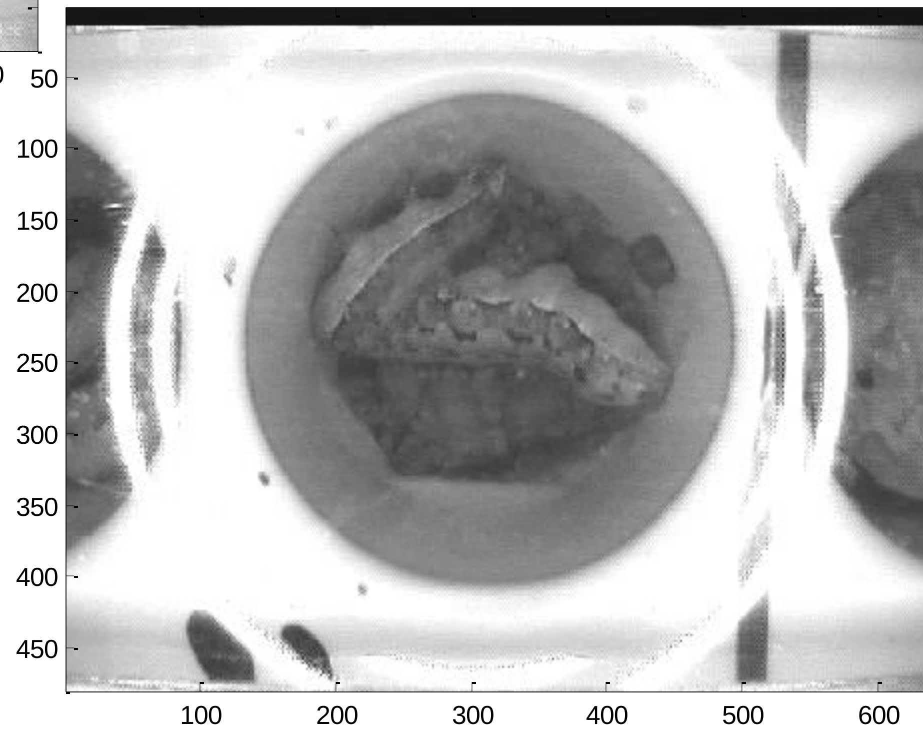
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Summary

- Most brains have lots of neurons
- Perceptrons (one-layer networks) insufficiently expressive
- Multi-layer networks are sufficiently expressive; can be trained by gradient descent, i.e., error back-propagation
- Many applications: speech, driving, handwriting, fraud detection, etc.
- Engineering, cognitive modeling, and neural system modeling subfields have largely diverged